RELATIONS

Relations & Functions

- Relation & function overview
- Cartesian Products of sets
- Relations
- Functions
- Function in terms of relation
- Real function & real-valued function
- Graphs
- Algebra of functions

Relation -

- A connection between or among things.
- E.g. Father & son is a relation , Brother & sister is a relation, student & teacher.

Note:

-Every relation has a '*pattern or property*'.

-Every relation involves '*minimum 2 identities*'.

Relation in mathematical world -

Examples –

- Number ‘p’ is greater than ‘q’.
- Line ‘m’ is perpendicular to line ‘n’
- Set A is a subset of set B.
- Relation between sides of a right triangle.

Function -

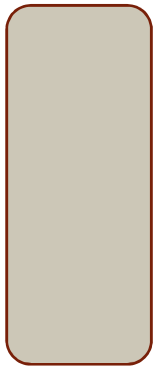
- Function is a set of action or activity.
- Visualize a function as a rule, which produces new elements out of some given elements.

E.g. Let police is function.

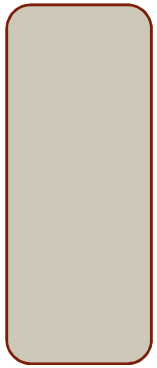
E.g. Teacher

Function in mathematical world-

- $F(x) = X^2$



- $F(x) = 2X$



Cartesian product of set -

- Suppose we have 3 shirts(green, blue, red) & 2 pants(black , blue).
- We can pair them as {(green, black), (green, blue), (blue, black), (Blue, blue), (red, black), (red , blue)} – 6 pairs

Given two non-empty sets P and Q.

- The Cartesian product $P \times Q$ is the set of all ordered pairs whose first component is a member of 'P' & second component is the member of 'Q'.

Remarks

(i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

(ii) $A \times B \neq B \times A$

(iii) $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

(iv) $n(A \times B) = n(A)*n(B)$; $n(A \times B \times C) = n(A)*n(B)*n(C)$

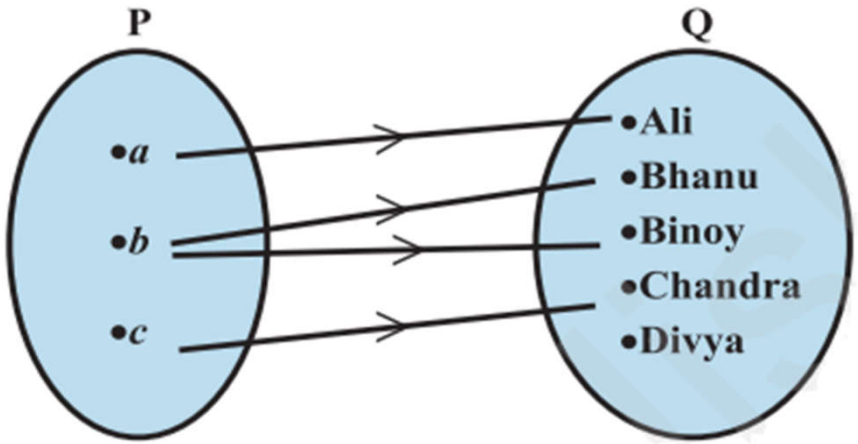
(v) If A x {infinite set} = {infinite set} where A is non-empty set.

E.g's

Relation – Some new terms

- Consider the two sets P = {a, b, c} and Q = {Ali, Bhanu, Binoy, Chandra, Divya}.
- The Cartesian product of P and Q has 15 ordered pairs.
- We now define a relation R,
- R= { (x,y): x is the first letter of the name y, $x \in P$, $y \in Q$}.
- R = {(a, Ali), (b, Bhanu), (b, Binoy), (c, Chandra)}

A visual representation of this relation R is called an arrow diagram



- Image -

  The second element in the ordered pair is called the image of the first element.

  E.g. Ali, bhanu, binoy, Chandra; **not** divya

- Domain –

  The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

  E.g. a , b , c

- Range-

  The set of all second elements in a relation R from a set A to a set B is called the range of the relation R.

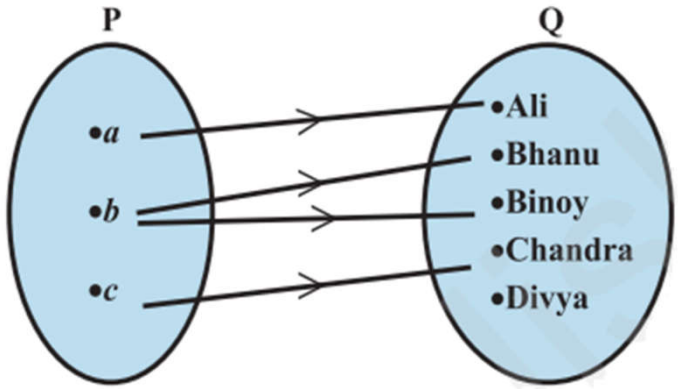
  E.g. Ali, bhanu, binoy, Chandra; **not** divya

- Co-domain –

  The whole set B is called the codomain of the relation R.

  E.g. Ali, bhanu, binoy, Chandra **and** divya

Note - range $\subseteq$ codomain



Functions -

- A special type of relation called function.
- Visualize a function as a rule, which produces new elements out of some given elements.
- There are many terms such as ‘map’ or ‘mapping’ used to denote a function.
- A relation f from a set A to a set B is said to be a function if every element of set A has 1 and only 1 image in set B.
- If f is a function from A to B and $(x,y) \in f$, then $f(x) = y$, where ‘y’ is called the image of a under f and ‘x’ is called the pre-image of ‘y’ under function ,f.

E.g. Test whether relation is a function or not?

- (i) R = {(2,1),(3,1), (4,2)},
- (ii) R = {(2,2),(2,4),(3,3), (4,4)}
- (iii) R = {(1,2),(2,3),(3,4), (4,5), (5,6), (6,7)}

E.g. Let **N** be the set of natural numbers and the relation R be defined on N such that R = {(x, y) : y = 2x, (x, y) $\in$ **N**}. What is the -

1.Domain,

2.Codomain and

3.Range of R?

Is this relation a function?

Real function & Real-valued function-

- Real valued function -

A function which has either R or one of its subsets as its range is called a real valued function.

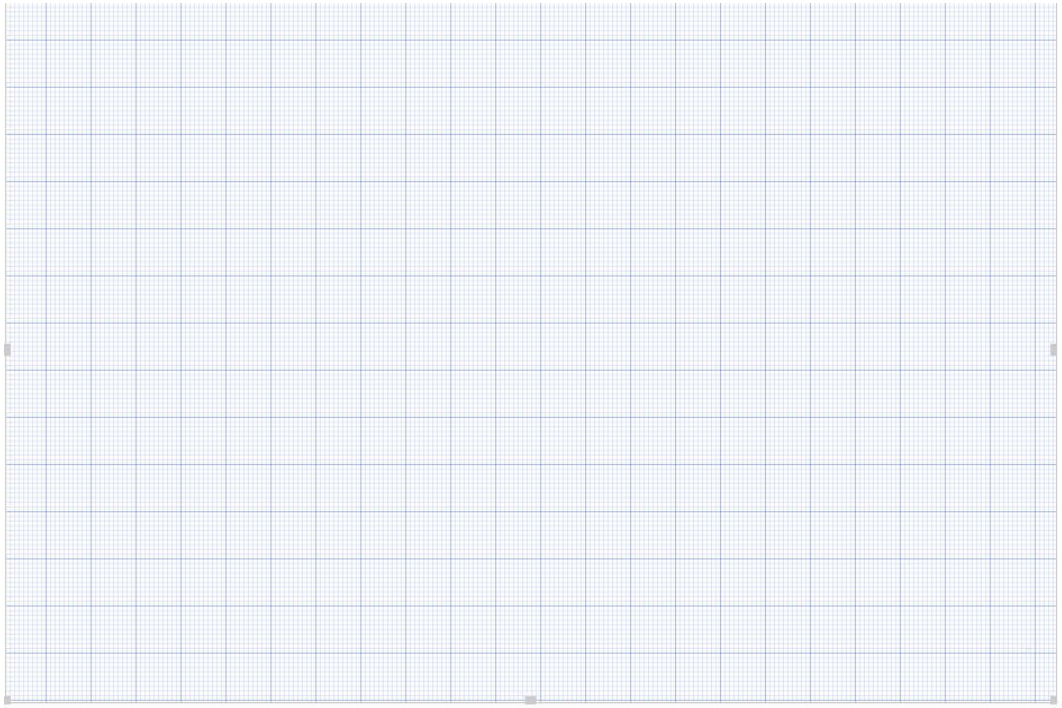
- Real function -

If its domain is also either R or a subset of R, it is called a real function

Functions – Graphs

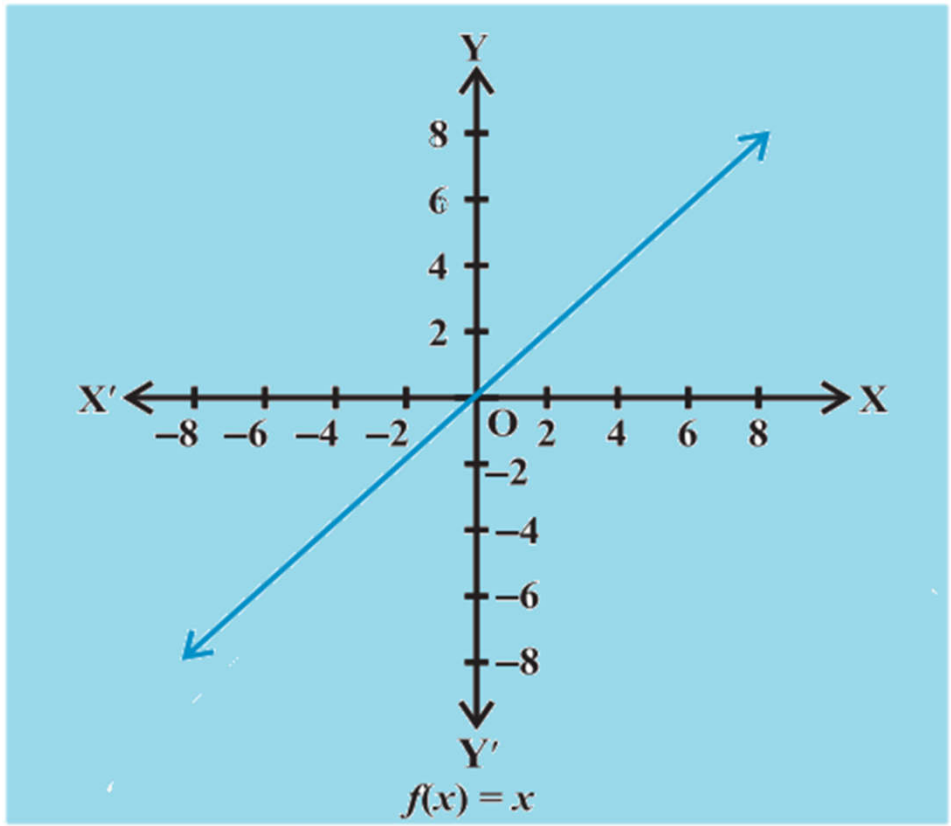
1. Polynomial function -

- A function f: R $\rightarrow$ R is said to be polynomial function if for each x in R, $y = f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where n is a non-negative integer and $a_0, a_1, a_2 \ldots a_n \in R$
- Domain = R & Range = Depends on f(x)
- The graph Depends on f(x).



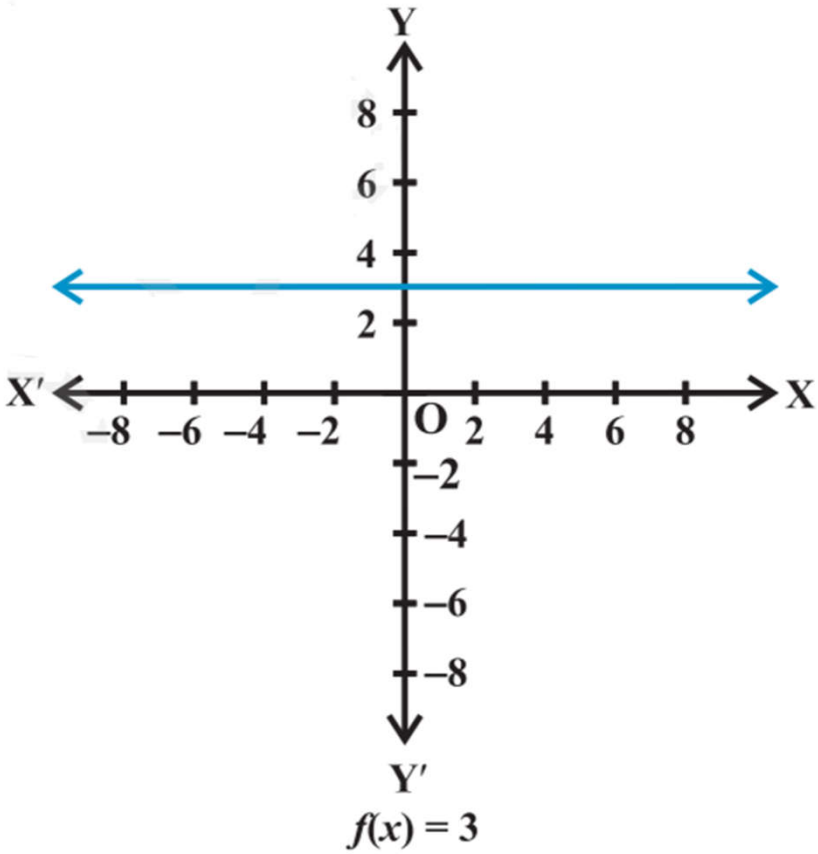
2.Identity function -

- F(x) = x
- Domain = R & Range = R
- The graph is a straight line.



3. Constant function –

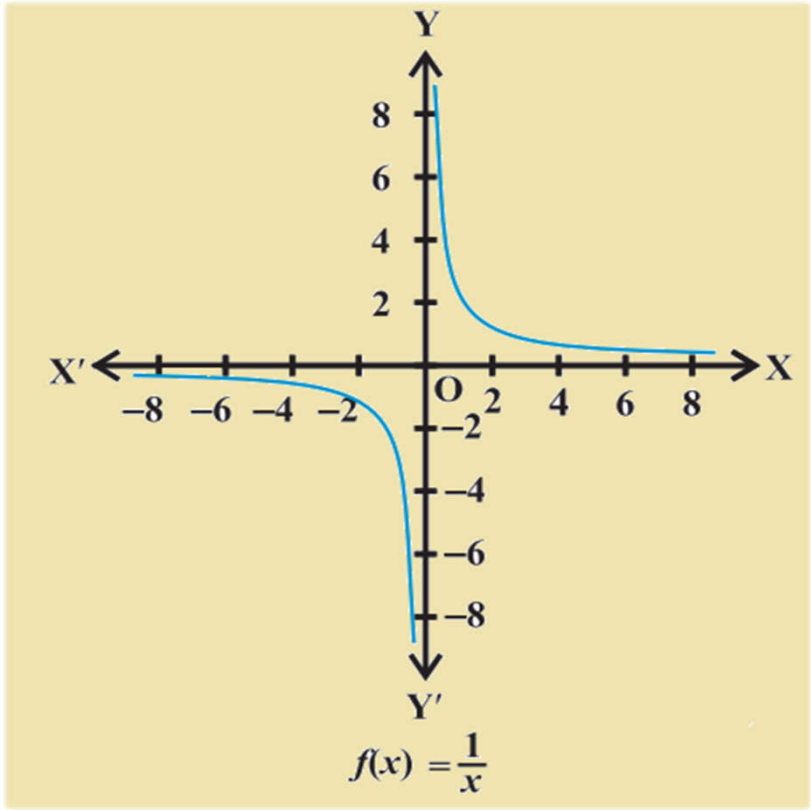
- F(x) = k , where ‘k’ is a constant
- Domain = R & Range = R
- The graph is a line parallel to x-axis



4. Rational function –

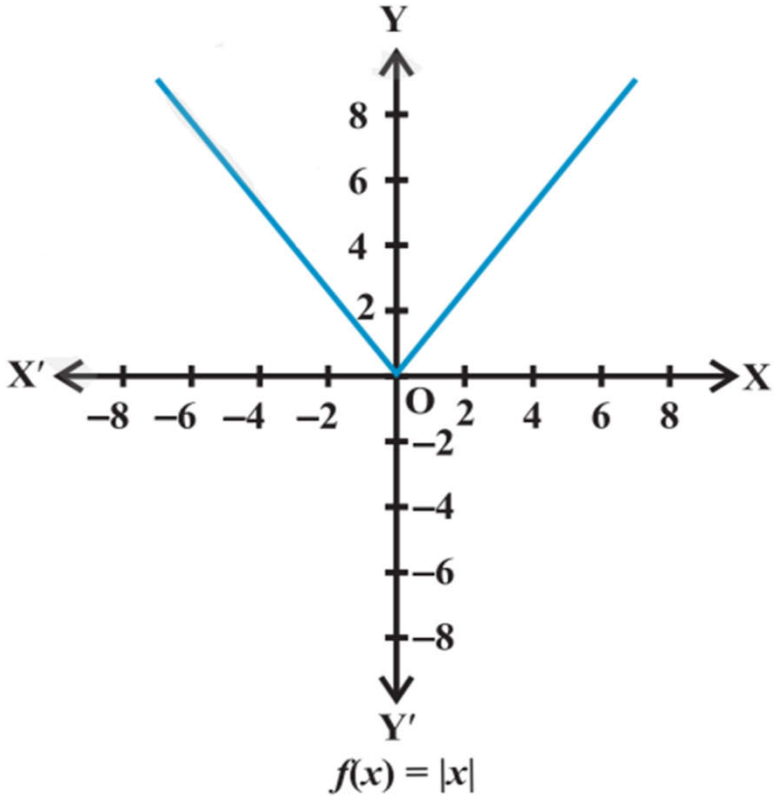
- f(x)/g(x); where f(x) & g(x) are polynomials function and g(x) ≠ 0
- Domain & Range both depend on f(x).

E.g. f(x) = 1/x



5. The Modulus function -

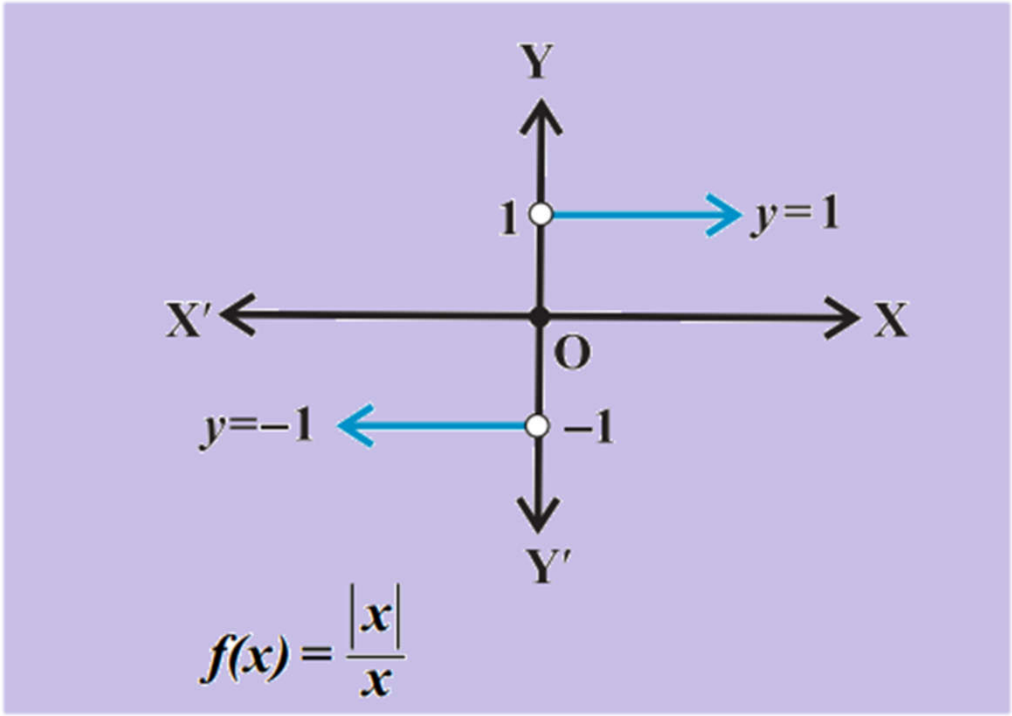
- $f(x) = |x|$
- Domain = R & Range = R+
- Graph is V-shaped



6. Signum function –

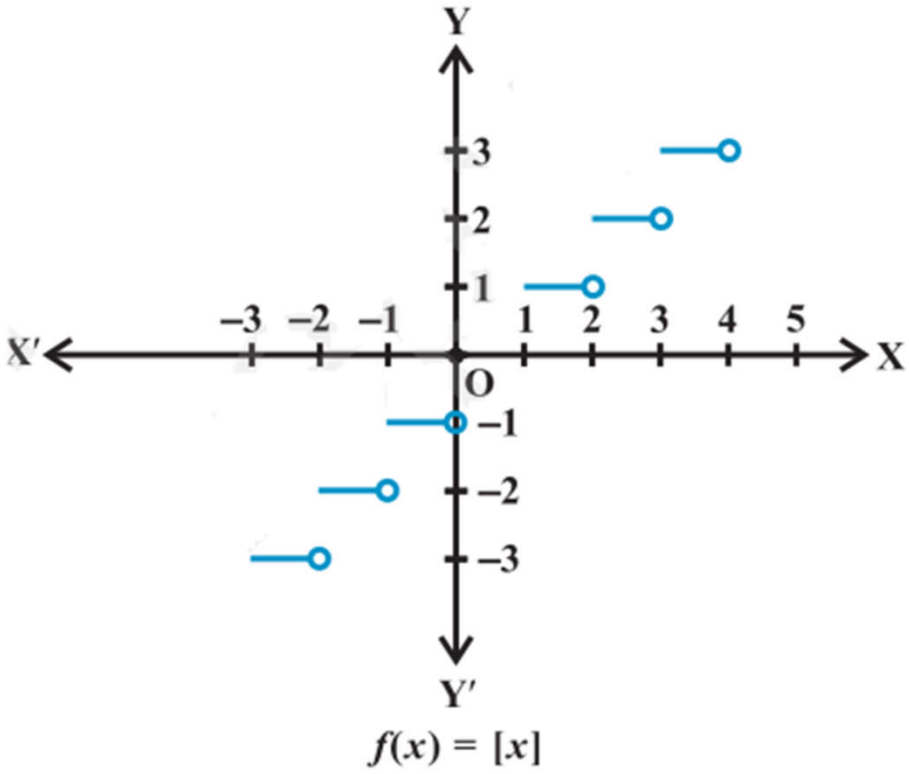
$$f(x)=\begin{cases}1, \text{if } x>0\\ 0, \text{if } x=0\\ -1, \text{if } x<0\end{cases}$$

- Domain = R & Range = {-1,0,1}



7. Greatest integer function –

- f(x) = [x]
- Domain = R & Range = N
- [x] = -1 for $-1 \leq x < 0$
- [x] = 0 for $0 \leq x < 1$
- [x] = 1 for $1 \leq x < 2$



Algebra of functions For functions

For functions $f : X \to R$ and $g : X \to R$, we have

- $(f + g)(x) = f(x) + g(x), x \in X$
- $(f – g)(x) = f(x) – g(x), x \in X$
- $(f.g)(x) = f(x).g(x), x \in X$
- $(k.f)(x) = k\, f(x)), x \in X$, where k is a real number.

End ☺